

Thermodynamic analysis of universes with the initial and final de-Sitter eras

H. Moradpour*, M. T. Mohammadi Sabet†, A. Ghasemi

Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran

Our aim is studying the thermodynamics of cosmological models including initial and final de-Sitter eras. For this propose, bearing Cai-Kim temperature in mind, we investigate the thermodynamic properties of a dark energy candidate with variable energy density, and show that the state parameter of this dark energy candidate should obey the $\omega_D \neq -1$ constraint, whiles there is no interaction between the fluids filled the universe, and the universe is not in the de-Sitter eras. Additionally, based on thermal fluctuation theory, we study the possibility of inducing fluctuations to the entropy of the dark energy candidate due to a mutual interaction between the cosmos sectors. Therefore, we find a relation between the thermal fluctuations and the mutual interaction between the cosmos sectors, whiles the dark energy candidate has a varying energy density. We point to models in which a gravitationally induced particle production process leads to change the expansion eras, whiles the corresponding pressure is considered as the cause of current acceleration phase. We study its thermodynamics, and show that such processes may also leave thermal fluctuations into the system. We also find an expression between the thermal fluctuations and the particle production rate. Finally, we use Hayward-Kodama temperature to get a relation for the horizon entropy in models including the gravitationally induced particle production process. Our study shows that the first law of thermodynamics is available on the apparent horizon whiles, the gravitationally induced particle production process, as the dark energy candidate, may add an additional term to the Bekenstein limit of the horizon.

I. INTRODUCTION

The universe expansion is modeled by the so-called FRW metric

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (1)$$

describing a homogenous and isotropic cosmos. In this metric, $a(t)$ is the scale factor while $k = -1, 0, 1$ is the curvature parameter corresponding to open, flat and closed universes respectively. WMAP data indicates a flat universe [1]. Conformal form of this metric can be used to model the inhomogeneity of cosmos due to the structure abundance in scales smaller than 100 Mpc [2]. The universe expansion, in the standard cosmology (Λ CDM), consists of four eras: (i) the primary inflation era ($a(t) \sim e^{Ht}$) exactly began after the big bang needed to remove the horizon problem. (ii) the radiation dominated era ($a(t) \sim t^{\frac{1}{2}}$) finished by leaving an attractive trace which is now called either CMB or LSS. (iii) the matter dominated era ($a(t) \sim t^{\frac{2}{3}}$) which has the major part in the structure formation process. (iv) finally, in agreement with the current phase of the expansion, the universe is undergoing an accelerated phase ($a(t) \sim e^{Ht}$) which is similar to the primary inflation era, where $H = \frac{\dot{a}}{a}$ and dot are the Hubble parameter and derivative with respect to time, respectively. In order to explain both the primary inflation and the current phase of the expansion leading to the coincidence and cosmological constant problems [1] we need an abnormal matters. Thermodynamic considerations show that the universe maintains this current phase of the expansion forever leading to avoid the big crunch problem [3]. We should note that this model (Λ CDM) is in a very good agreement with the observations [1, 4–8]. It should be noted that if one accepts the thermodynamic predictions about the current stage of the expansion [3], then three questions including the big bang, coincidence and cosmological constant problems remain unsolved.

There are various models proposed to eliminate such weaknesses of the standard cosmology by introducing either a new degree of freedom or a new parameter leading to explanations for the generator of the current acceleration phase (called dark energy (DE)) in the Einstein relativity framework [9–18]. Moreover, it is shown that the DE candidates may add an additional term to the Bekenstein entropy of the apparent and trapping horizons of the FRW universe [19–21]. Recently, two models are introduced in the literatures to solve the mentioned problems of the standard cosmology whenever their physics are completely different from those of previous works [22, 23]. In these models, the universe expansion is began from an unstable de-Sitter spacetime leading to solve the horizon problem. In addition

* h.moradpour@riaam.ac.ir

† mohamadisabet@shirazu.ac.ir

to this, the final stage of the universe expansion is an eternal de-Sitter phase which is due to the thermodynamic equilibrium conditions and leading to solve the big crunch singularity problem [24, 25]. Because of different physics behind these models, they become interesting for further investigations [26–32]. As the main difference of these two models, while a varying vacuum plays the role of DE in the model proposed in [22], the gravitationally induced particle production process is the backbone of the second proposal [23, 25]. Moreover, it is shown that the first model satisfies the Bekenstein limit of entropy [21].

It seems that the apparent horizon of the FRW universe, as a marginally trapped surface which is located at [33, 34]

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a(t)^2}}}, \quad (2)$$

can be considered as a causal boundary for this spacetime [35–40]. Surface gravity of the apparent horizon is evaluated as

$$\kappa = \frac{1}{2\sqrt{-h}} \partial_a (\sqrt{-h} h^{ab} \partial_b \zeta), \quad (3)$$

leading to

$$\kappa = -\frac{H}{2\pi} \left(1 + \frac{\dot{H}}{2H^2}\right), \quad (4)$$

and we get

$$T = \frac{\kappa}{2\pi} = -\frac{H}{2\pi} \left(1 + \frac{\dot{H}}{2H^2}\right), \quad (5)$$

as the temperature on the apparent horizon [33, 34]. Indeed, this temperature is called Hayward-Kodama temperature used to show the consistency between the Friedmann equations and the first law of thermodynamics [40–43]. Moreover, some authors define $T = \frac{|\kappa|}{2\pi} \simeq \frac{H}{2\pi}$, called Cai-Kim temperature, in order to get the positive value for temperature [44]. Another motivation for $T = \frac{H}{2\pi}$, signalling us that this temperature may be considered as the temperature for fields confined to the apparent horizon, can be found in ref. [45]. Finally, it is useful to note that these definitions of temperature could not attract a common agreement [41–43].

Bearing the various definitions of temperature in mind, since the apparent horizon can be considered as the causal boundary, some authors have been shown that the validity of the first law of thermodynamics on the apparent horizon leads to Friedmann equations [40–52]. In addition, it seems that it is necessary to consider a DE [1, 53, 54] or modifying the Einstein equations [55] in order to be compatible with recent observations imposing the $\dot{a}(t) \geq 0$ and $\ddot{a}(t) \geq 0$ conditions on the scale factor [56–59]. These data are also in agreement with the generalized second law of thermodynamics in numerous models of DE providing an eternal thermodynamic equilibrium state for the universe [3, 24, 60–62]. More studies on the thermodynamics of final state of the cosmos can be found in ref. [63, 64].

Moreover, observations admit an interaction between the dark sectors of the cosmos [65–71]. Additionally, it seems that the mutual interaction between the dark sectors of the universe may solve the coincidence problem [71–78]. Considering thermal fluctuation theory in mind [79], authors have shown that the entropy of event horizon is modified by a logarithmic correction [80]. In the cosmological setup, by using the Cai-Kim temperature, it is shown that these fluctuations may be interpreted as the result of a mutual interaction between the dark components [78]. The generalization of this approach to more cosmological models can be found in [81–86]. Therefore, it seems that one can find an expression for the mutual interaction between the dark sectors of the cosmos by taking into account the thermal fluctuations of the universe components. In addition, it is shown that a mutual interaction between the DE candidate and the other parts of cosmos may add an additional term to the Bekenstein entropy of the apparent horizon [21].

Here, according to the foregoing discussion and by following the approach considered in refs. [78, 81–86], we focus on the models proposed in Refs. [22, 23], and try to find the suitable thermodynamic interpretations for these models by using the Cai-Kim temperature as well as the thermal fluctuation theory. In fact, we try to get a thermodynamic interpretation for the mutual interaction between the DE candidate and the other parts of cosmos. We also use the Hayward-Kodama temperature, and show that the gravitationally induced particle production process, as the DE candidate, may add an additional term to the Bekenstein entropy of the apparent horizon. The latter is in agreement with the previous attempts which predict that the DE candidates may correct the Bekenstein bound in the cosmological setups [19–21]. For simplicity, we take $G = \hbar = c = 1$ throughout this paper and we restrict ourselves to the $k = 0$ case. Moreover, dot denotes derivative with respect to time.

The paper is organized as follows. In next section, we decompose the universe sectors into a varying DE candidate and other parts with total density ρ and state parameter ω whiles, the cosmos sectors do not interact with each other. Then, we study the thermodynamics of the DE candidate in this model. In section III, we consider the model introduced in [22], and give a thermodynamical interpretation for this model using thermal fluctuations theory. To do this, the model will be considered as a model in which the cosmos sectors are interacting with each other. In section IV, we focus on the model proposed in Ref. [23], and show that the gravitationally induced particle production process leads to the thermal fluctuations and finally, we get a thermodynamical description for this model. We also show that the gravitationally induced particle production process may add an additional term to the Bekenstein limit of the apparent horizon of the flat FRW universe in section V. Last section is devoted to summary and concluding remarks.

II. THERMODYNAMICAL DESCRIPTION OF DE MODELS WITH NON-CONSTANT ENERGY DENSITY

For the flat FRW universe supported a DE candidate, Friedmann equations lead to

$$H^2 = \frac{8\pi}{3}(\rho + \rho_D), \quad (6)$$

and

$$-2\frac{\ddot{a}}{a} - H^2 = 8\pi G(p + p_D) \quad (7)$$

where ρ_D and p_D are the density of DE and its corresponding pressure, respectively. In addition, ρ comprises other parts of cosmos which may include radiation, pressureless matter, dark matter and etc. p is also the pressure corresponding to the density ρ . Consider a DE candidate with profile density

$$\rho_D(H) = \frac{\Lambda(H)}{8\pi} = \alpha + \beta H^2 + \gamma H^{2n}, \quad (8)$$

which converges to that of the ghost dark energy model by substituting $\alpha = \beta = 0$ and $n = \frac{1}{2}$ [9]. Moreover, it covers the profile density of generalized ghost dark energy model by inserting $\alpha = 0$ together with $n = \frac{1}{2}$ [17]. The cosmological constant model, as the trivial limit, is obtainable by inserting $\beta = \gamma = 0$ [1]. More models in which authors used a dynamic DE model to explain the current expanding phase can be achieved by choosing proper values for n , α , β and γ [22, 87–93]. It is also shown that this profile density may add an additional term to the Bekenstein entropy of the trapping and apparent horizons of the FRW universe [19–21]. Using this equation, one gets

$$\rho = H^2\left(\frac{3}{8\pi} - \beta\right) - \alpha - \gamma H^{2n}. \quad (9)$$

For the total energy momentum tensor, the energy momentum conservation law implies

$$\dot{\rho} + \dot{\rho}_D + 3H(\rho(1 + \omega) + \rho_D(1 + 3\omega_D)) = 0, \quad (10)$$

where dot denotes derivative with respect to time. This equation can be decomposed into

$$\dot{\rho} + 3H\rho(1 + \omega) = 0, \quad (11)$$

and

$$\dot{\rho}_D + 3H\rho_D(1 + \omega_D) = 0, \quad (12)$$

where $\omega_i = \frac{p_i}{\rho_i}$ and p_i are the state parameter and the pressure of the i^{th} sector, respectively. Such well-advised decompositions are valid whenever there is no mutual interaction between the dark sectors. In such a situation, from Eqs. (8) and (12), clearly we have $\dot{\rho}_D \neq 0$ yielding $\omega_D \neq -1$. For models introduced in Ref. [22], the energy profile density of varying vacuum, as the DE candidate, is also given by (8), whiles $\omega_D = -1$, and satisfies the thermodynamic equilibrium conditions in current acceleration phase of the universe expansion [24]. It is shown that such models may avoid the big bang and big crunch singularities as well as the horizon problem, and can also provide a complete description for the history expansion of the universe [22]. We should note here that since the state parameter of the DE candidate satisfies the $\omega_D = -1$ constraint in model proposed by Lima et al. [22], the decomposition of (10)

into (11) and (12) is possible if $\dot{\rho} = 0$ leading to $\dot{H} = 0$ because $\dot{\rho} = \frac{d\rho}{dH}\dot{H}$. Briefly, since model proposed by Lima et al. [22] satisfies the thermodynamic equilibrium conditions in the current de-Sitter accelerating phase ($\dot{H} = 0$) [24], the decomposition of (10) into Eqs. (11) and (12) as the result of marginally thermodynamic equilibrium states (de-Sitter spacetimes with $\dot{H} = 0$) of the model is reasonable.

Derivation from Eq. (6) with respect to t and using (10), leads to (Raychaudhuri equation)

$$\dot{H} = \frac{dH}{dt} = -4\pi[\rho(1 + \omega) + \rho_D(1 + \omega_D)]. \quad (13)$$

Now, if we define $\rho_c \equiv 3\frac{H^2}{8\pi}$ and use Eqs. (6), (8) and (10) we obtain

$$\dot{H} = -4\pi[(\omega_D - \omega)(\alpha + \beta H^2 + \gamma H^{2n}) + \frac{3H^2}{8\pi}(1 + \omega)]. \quad (14)$$

Since we have only used Eqs. (6–10) in order to derive Eqs. (13) and (14), we should note that these equations are independent of the validity of Eqs. (11) and (12), and thus the probable mutual interaction between the dark sectors. Indeed, the validity of equation (13), and therefore (14), is due to the Bianchi identity or the conservation of the total energy momentum tensor (10). Finally, by inserting ρ_c into the Friedmann equation we get

$$1 = \Omega_D + \Omega, \quad (15)$$

where $\Omega_i = \frac{\rho_i}{\rho_c}$ is the fractional energy density of the i^{th} component of the cosmos. For the DE candidate confined to the flat FRW universe enclosed by the apparent horizon (2), the Gibb's law implies

$$TdS_D = dE_D + p_D dV. \quad (16)$$

In this equation, S_D is associated entropy to the DE while $V = \frac{4\pi}{3}\tilde{r}_A^3$ and $E_D = \rho_D V$ are the volume of the flat FRW universe and the energy of DE, respectively. Additionally, thermodynamic equilibrium condition implies that T (the temperature of DE) should has the same value as the temperature of the apparent horizon. Moreover, since it is unreasonable to have a fluid with negative temperature, meaning that $T > 0$, the Cai-Kim temperature may be a good option for the temperature of the DE candidate [34, 39, 42, 44, 46, 50, 78, 81–86]

$$T = \frac{H}{2\pi} = \frac{1}{2\pi\tilde{r}_A}, \quad (17)$$

where \tilde{r}_A is the apparent horizon radius of the flat FRW universe (2)

$$\tilde{r}_A = \frac{1}{H}. \quad (18)$$

Therefore, for the volume and energy differentials we reach

$$dV = 4\pi(\tilde{r}_A)^2 d\tilde{r}_A = -4\pi H^{-4} dH, \quad (19)$$

and

$$dE_D = \rho_D dV + V d\rho_D, \quad (20)$$

leading to

$$dS_D = 2\pi\tilde{r}_A(\rho_D(1 + \omega_D)4\pi(\tilde{r}_A)^2 d\tilde{r}_A + \frac{4\pi}{3}(\tilde{r}_A)^3 d\rho_D), \quad (21)$$

where we have used $p_D = \rho_D \omega_D$. In addition, since

$$\dot{\rho}_D = \frac{d\rho_D}{d\tilde{r}_A} \frac{d\tilde{r}_A}{dH} \frac{dH}{dt}, \quad (22)$$

we find

$$\frac{d\rho_D}{d\tilde{r}_A} = -\frac{3\rho_D(1 + \omega_D)}{4\pi\tilde{r}_A^3(\rho(1 + \omega) + \rho_D(1 + \omega_D))}, \quad (23)$$

where we have used Eqs. (12), (13) and (18). By combining this equation with (21) and after some algebra, we obtain

$$dS_D^0 = 8\pi^2 \rho_D^0 (1 + \omega_D^0) (\tilde{r}_A^0)^3 \left(1 - \frac{1}{4\pi (\tilde{r}_A^0)^2 (\rho^0 (1 + \omega^0) + \rho_D^0 (1 + \omega_D^0))}\right) d\tilde{r}_A^0. \quad (24)$$

We must note that the superscript (0) is used to indicate that this result is valid whenever there is no interaction between the cosmos sectors of the universe and therefore, $\tilde{r}_A^0 = \frac{1}{H_0}$. Since we did not use Eq. (8) to obtain this equation, this result (Eq. (24)) is also valid in every cosmological model with the same Friedmann equation as Eq. (6). Using Eqs. (8), (14) and (18) the above equation can be rewritten as follows

$$dS_D^0 = -\frac{8\pi^2 (\alpha + \beta H_0^2 + \gamma H_0^{2n}) (1 + \omega_D^0)}{H_0^5} \left(1 - \frac{H_0^2}{4\pi [(\omega_D^0 - \omega^0)(\alpha + \beta H_0^2 + \gamma H_0^{2n}) + \frac{3H_0^2}{8\pi} (1 + \omega^0)]}\right) dH_0. \quad (25)$$

Now, using Eqs. (8) and (15) to get

$$\frac{dS_D^0}{dH_0} = \frac{3\pi \Omega_D^0 (1 + \omega_D^0)}{H_0^3} \left(\frac{2}{3[(\omega_D^0 - \omega^0)\Omega_D^0 + (1 + \omega^0)]} - 1\right), \quad (26)$$

where the superscript/subscript (0) implies non-interacting case. Therefore, we find an expression for the entropy of the DE candidate when its energy density profile is varying with time as introduced in Eq. (8). It is useful to be noted that, in the $\omega_D^0 = -1$ limit, $dS_D^0 = 0$ is obtainable meaning that $\dot{\rho}_D = 0$ which can be considered as either the cosmological constant model of DE [1], or the marginally thermodynamic equilibrium states of model proposed by Lima et al. [22]. In fact, from (26) it is apparent that, while $\omega_D^0 = -1$, $\frac{dS_D^0}{dH_0} = \frac{d^2 S_D^0}{dH_0^2} = 0$ is available, meaning that the thermodynamic equilibrium conditions are marginally satisfied in these eras [94]. The latter signals us that the initial and final de-Sitter spacetimes of models proposed by Lima et al. [22] are marginally thermodynamic equilibrium states. In the next section, we show that whenever a mutual interaction between the cosmos sectors moves the cosmos between these two marginally thermodynamic equilibrium states [22], it leaves a thermal fluctuations into the system.

III. THERMODYNAMIC DESCRIPTION OF THE INTERACTING DE MODELS WITH NON-CONSTANT ENERGY DENSITY

Here, we study the thermodynamics of a universe filled by a varying vacuum, as the DE candidate, together with another source of energy with density ρ interacting with each other whenever the energy density of the DE candidate is the same as the previous section (8). In this manner, decomposition of the energy-momentum conservation law (10) implies

$$\dot{\rho} + 3H\rho(1 + \omega) = -Q, \quad (27)$$

and

$$\dot{\rho}_D + 3H\rho_D(1 + \omega_D) = Q, \quad (28)$$

where Q is the mutual interaction between the cosmos sectors. Clearly, $\omega_D = -1$ is accessible in this model while $\dot{\rho}_D \neq 0$. Some authors have considered this possibility ($\omega_D = -1$), and showed that the model leads to the compatible outcomes with observational data and the thermodynamic equilibrium conditions [22, 24]. In addition, it is also shown that, independent of the profile density of the DE candidate, the apparent horizon satisfies the Bekenstein entropy, in the interacting models with $\omega_D = -1$ [21]. It seems that the mutual interaction between the dark sectors induce some fluctuations into the thermodynamic properties of systems which can be investigated by thermal fluctuations theory [78, 80–86]. Due to these fluctuations, the entropy is changed to S_D and it can be expanded as [78–86]

$$S_D = S_D^0 + S_D^1 + S_D^2 \quad (29)$$

In this equation, S_D^0 is the entropy of the DE candidate when there is no mutual interaction between the cosmos sectors of the universe and $S_D^1 = -\frac{1}{2} \ln CT_0^2$ is logarithmic correction to the entropy where $C = T_0 \frac{dS_D^0}{dT_0}$ is the dimensionless heat capacity. S_D^2 also concerns higher order terms. It is also useful to mention that this analysis is valid for all thermodynamical systems [80]. Combining Eqs. (24) and (17), we get

$$C = T_0 \frac{dS_D^0}{dT_0} = -\frac{\rho_D^0 (1 + \omega_D^0)}{T_0^4} \left(1 - \frac{\pi T_0^2}{\rho^0 (1 + \omega^0) + \rho_D^0 (1 + \omega_D^0)}\right). \quad (30)$$

From Eq. (17) we have also

$$C = T_0 \frac{dS_D^0}{dT_0} = H_0 \frac{dS_D^0}{dH_0}, \quad (31)$$

leading to

$$C = -\frac{8\pi^2(\alpha + \beta H_0^2 + \gamma H_0^{2n})(1 + \omega_D^0)}{H_0^4} \left(1 - \frac{H_0^2}{4\pi[(\omega_D^0 - \omega^0)(\alpha + \beta H_0^2 + \gamma H_0^{2n}) + \frac{3H_0^2}{8\pi}(1 + \omega^0)]}\right), \quad (32)$$

where we have used Eq. (25) in order to evaluate the heat capacity of the DE candidate with profile density (8). Therefore, one gets

$$S_D^1 = -\frac{1}{2} \ln\left(\left[\frac{32\pi^4(\alpha + \beta H_0^2 + \gamma H_0^{2n})(1 + \omega_D^0)}{H_0^2} \left(\frac{H_0^2}{4\pi[(\omega_D^0 - \omega^0)(\alpha + \beta H_0^2 + \gamma H_0^{2n}) + \frac{3H_0^2}{8\pi}(1 + \omega^0)]} - 1\right)\right]\right). \quad (33)$$

In deriving this equation we used again Eqs. (17) and (18) along as (32). It is a matter of calculation to show that

$$\begin{aligned} \frac{dS_D^1}{dH_0} = & -\frac{1}{2} \left(\frac{2\beta H_0 + 2n\gamma H_0^{2n-1}}{\alpha + \beta H_0^2 + \gamma H_0^{2n}} + \frac{2H_0 - 4\pi[(\omega_D^0 - \omega^0)(2\beta H_0 + 2n\gamma H_0^{2n-1}) + \frac{3H_0}{4\pi}(1 + \omega^0)]}{H_0^2 - 4\pi[(\omega_D^0 - \omega^0)(\alpha + \beta H_0^2 + \gamma H_0^{2n}) + \frac{3H_0^2}{8\pi}(1 + \omega^0)]} \right. \\ & \left. - \frac{[(\omega_D^0 - \omega^0)(2\beta H_0 + 2n\gamma H_0^{2n-1}) + \frac{3H_0}{4\pi}(1 + \omega^0)]}{[(\omega_D^0 - \omega^0)(\alpha + \beta H_0^2 + \gamma H_0^{2n}) + \frac{3H_0^2}{8\pi}(1 + \omega^0)]} - \frac{2}{H_0} \right), \end{aligned} \quad (34)$$

which can be simplified, by using Eq. (8), as

$$\frac{dS_D^1}{dH_0} = -\frac{1}{2} \left(\frac{(\rho_D^0)'}{\rho_D^0} + \frac{2H_0 - 4\pi[(\omega_D^0 - \omega^0)(\rho_D^0)' + \frac{3H_0}{4\pi}(1 + \omega^0)]}{H_0^2 - 4\pi[(\omega_D^0 - \omega^0)\rho_D^0 + \frac{3H_0^2}{8\pi}(1 + \omega^0)]} - \frac{[(\omega_D^0 - \omega^0)(\rho_D^0)' + \frac{3H_0}{4\pi}(1 + \omega^0)]}{[(\omega_D^0 - \omega^0)\rho_D^0 + \frac{3H_0^2}{8\pi}(1 + \omega^0)]} - \frac{2}{H_0} \right), \quad (35)$$

where the prime stands for the derivative with respect to H_0 . Again, we should note that the subscript/superscript (0) is used to emphasize the non-interacting parameters. By using Friedmann equation (6) along as Eqs. (18) and (28), it is easy to show that

$$\frac{d\rho_D}{d\tilde{r}_A} = \frac{Q - 3H\rho_D(1 + \omega_D)}{4\pi\tilde{r}_A^2(\rho(1 + \omega) + \rho_D(1 + \omega_D))}. \quad (36)$$

By following the recipe of previous section we get

$$dS_D = 8\pi^2\rho_D(1 + \omega_D)(\tilde{r}_A)^3 \left(1 + \frac{Q - 3H\rho_D(1 + \omega_D)}{12\pi\tilde{r}_A\rho_D(1 + \omega_D)(\rho(1 + \omega) + \rho_D(1 + \omega_D))}\right) d\tilde{r}_A. \quad (37)$$

Using Eq. (18), we find

$$\frac{dS_D}{dH} = \frac{8\pi^2\rho_D(1 + \omega_D)}{H^5} \left(\frac{3H^2\rho_D(1 + \omega_D) - QH}{12\pi\rho_D(1 + \omega_D)(\rho(1 + \omega) + \rho_D(1 + \omega_D))} - 1\right). \quad (38)$$

As a desired result, the results of previous section are obtainable by substituting $Q = 0$. Bearing the model investigated in [22, 24] in mind, where $\omega_D = -1$, and use Eq. (9) to obtain

$$\frac{dS_D}{dH} = -\frac{2\pi Q}{3(1 + \omega)H^4(\alpha + \gamma H^{2n} - H^2(\frac{3}{8\pi} - \beta))}. \quad (39)$$

Comparing this equation together with (26), we can conclude that the mutual interaction may change the entropy of the varying vacuum leading to separation from the marginally thermodynamic equilibrium situation ($dS_D^0 = d^2S_D^0 = 0$). Since $Q = \dot{\rho}_D$ in this model, by using (8) and (14), we get

$$Q = 4\pi\rho_D'(\omega + 1)(\rho_D - \frac{3H^2}{8\pi}). \quad (40)$$

Considering the definition of the fractional energy density, this equation can be written as

$$Q = \frac{3}{2}\rho'_D(\omega + 1)H^2(\Omega_D - 1), \quad (41)$$

leading to

$$\frac{dS_D}{dH} = -\frac{8\pi^2\rho'_D}{3H^4}, \quad (42)$$

where again, prime stands for the derivative with respect to H . We should note that the results of either considering cosmological constant model of the DE or the initial and ultimate thermodynamic equilibrium states of Lima et al. model [22] (or briefly, $dS_D = 0$) are obtainable by inserting $\omega_D = -1$ and $Q = 0$ simultaneously. Since Eq. (29) implies

$$\frac{dS_D}{dH} = \left(\frac{dS_D^0}{dH_0} + \frac{dS_D^1}{dH_0} + \frac{dS_D^2}{dH_0}\right)\frac{dH_0}{dH}, \quad (43)$$

we use Eq. (39) to get

$$\frac{dS_D^2}{dH_0} = \frac{16\pi^2 Q \dot{H}}{9(1 + \omega_m)H^6(1 - \Omega_D)\dot{H}_0} - \frac{dS_D^0}{dH_0} - \frac{dS_D^1}{dH_0}, \quad (44)$$

where, $\frac{dS_D^0}{dH_0}$ and $\frac{dS_D^1}{dH_0}$ are introduced in Eqs. (26) and (35), respectively. We have also used (15) and $\frac{dH}{dH_0} = \frac{\dot{H}}{\dot{H}_0}$ to derive this equation, while \dot{H} and \dot{H}_0 are evaluated by using Eq. (14) for the interacting and non-interacting cases, respectively. Therefore, we find an expression for the thermal fluctuations which are due to the interaction $Q = \dot{\rho}_D$. Since it seems that the S_D^2 term has insignificant effects with respect to the S_D^1 term in the gravitational and cosmological setups [78, 80–86], we disregard it and inserting Eq. (42) into (43), to get

$$\frac{\rho'_D}{H^4}dH = -\frac{3(dS_D^0 + dS_D^1)}{8\pi^2}. \quad (45)$$

Now, using (8), we reach

$$\frac{\beta}{H^2} + \frac{\gamma}{\frac{4}{2n} - 1}H^{2(n-2)} = \frac{3}{8\pi^2}(S_D^0 + S_D^1 + C), \quad (46)$$

where C is an integration constant. Additionally, due to this fact that entropy is not an absolute quantity, C can be set to zero without loss of generality. S_D^1 is evaluated in Eq. (33), while S_D^0 can be obtained by integrating from Eq. (25). This is a relation for the Hubble parameter, up to the first order fluctuations, due to the interaction Q . Loosely speaking, based on the first order terms of the thermal fluctuations which are due to the interaction between the DE candidate and the other parts of cosmos, we find the mutual relation between H and H_0 in the model proposed in Ref. [22]. It should be noted that Eqs. (44–46) are only valid when $\omega_D = -1$, $Q = \dot{\rho}_D$ while ρ_D is explained by Eq. (8). One can also find a general relation between an unknown Q and thermal fluctuations in the models with arbitrary energy density for the DE, by using Eqs. (24), (30), (38) and (43).

IV. THERMODYNAMICAL DESCRIPTION OF GRAVITATIONALLY PARTICLE PRODUCTION INDUCED PROCESS

Here, we focus on the LBC model proposed in Ref. [23, 25]. In this model, like the previous model [22], the universe expansion is began from an unstable initial de-Sitter spacetime and follows the radiation and matter dominated era in continue. Finally, the universe expansion will reach to a perpetual de-Sitter phase which is in agreement with the thermodynamical equilibrium condition [24, 25]. In this model, there is a particle production due to the gravitational effects leading to an additional pressure to the Friedman equations as:

$$H^2 = \frac{8\pi}{3}\rho, \quad (47)$$

and

$$-2\frac{\ddot{a}}{a} - H^2 = 8\pi(p + p_C), \quad (48)$$

where we considered a flat background [23, 25]. ρ is the energy density of the dominated perfect fluid, such as the radiation, confined to the apparent horizon of the FRW universe whiles, p is the corresponding pressure. The additional pressure (p_C) plays the key role in the change of the expansion phase, and depends on the particle production rates. In a simple approach, considering an adiabatic process for the particle creation, the entropy per particle is constant whenever the total entropy is not [95]. Therefore, one reaches [95–100]

$$p_C = -\frac{\rho(1+\omega)\Gamma}{3H}, \quad (49)$$

whiles Γ is the particle production rate with dimension of $(\text{time})^{-1}$ [23]. the Energy-momentum conservation law implies

$$\dot{\rho} + 3H\rho(1+\omega) = -Q, \quad (50)$$

where $Q = 3Hp_C$ [23, 25]. In the absence of this particle production ($Q = 0$), the entropy of the fluid with density ρ , pressure p and the state parameter ω reads as

$$dS^0 = -\frac{\pi}{H_0^3}(1+3\omega^0)dH_0, \quad (51)$$

where we have followed the recipe of section II. Indeed, one can use Eqs. (24) and (18), substitute $\rho^0 = 0$, and finally replace ρ_D with ρ together with using (47) to get this equation. We should note that since for the cosmological constant model $\dot{\rho} = 0$ leading to $dH_0 = 0$, this equation covers the result of considering the cosmological constant model. We used the subscript/superscript (0) to indicate that there is no particle production in this situation. In addition, due to the universe expansion, the densities of confined fluids, including radiation and etc., are diluted. This provides a suitable situation in which the effects of this particle production process overcomes those of the other fluids leading to change the expansion phase. Bearing Eq. (47) in mind, in order to evaluate the effects of this pressure on the entropy of the dominated fluid, we use Eq. (38) while $\rho = 0$ to get

$$\frac{dS}{dH} = \frac{\pi(1+\omega)}{H^3} \left(\frac{18\pi H^4(1+\omega) - 16\pi QH}{3H^4(1+\omega)^2} - 1 \right), \quad (52)$$

where H is the Hubble parameter when the effects of the particle production is considered. In order to calculate the corresponding thermal fluctuations due to this pressure, by using Eq. (30), we get

$$CT^2 = -\frac{3(1+3\omega^0)}{4}, \quad (53)$$

leading to $dS^1 = 0$ for $\omega^0 = \text{cons}$. Since this result is independent of p_C , we can conclude that the gravitationally induced particle production processes induce weak fluctuations into the thermodynamic properties of the cosmos supported by a perfect fluid with constant state parameter. It explains that why these processes do not disturb the current thermodynamic equilibrium state of the cosmos investigated in [24, 25]. In order to find an expression for S^2 , one can insert Eqs. (51) and (52) into (29) as

$$\frac{dS^2}{dH_0} = \frac{dS}{dH} \frac{dH}{dH_0} - \frac{dS_0}{dH_0}, \quad (54)$$

which yields

$$\frac{dS^2}{dH_0} = \frac{\pi(1+\omega)}{H^3} \left(\frac{18\pi H^2 + 6\Gamma}{3H^2(1+\omega)^2} - 1 \right) \frac{dH}{dH_0} + \frac{\pi}{H_0^3}(1+3\omega^0), \quad (55)$$

for models obeying (49). It is easy to show that

$$\dot{H}_0 = -\frac{3}{2}H_0^2(1+\omega^0), \quad (56)$$

where $p_C = 0$, and

$$\dot{H} = -\frac{3}{2}H^2(1+\omega)\left[1 - \frac{\Gamma}{3H}\right], \quad (57)$$

while $p_C \neq 0$ and obeys (49). In deriving these equations (Raychaudhuri equations) we have used Eqs. (47) and (50). Now, since

$$\frac{dH}{dH_0} = \frac{\dot{H}}{\dot{H}_0}, \quad (58)$$

we get

$$\frac{dS^2}{dH_0} = \frac{\pi}{H_0^2} \left[-\frac{(1+\omega)^2}{H(1+\omega^0)} \left(\frac{18\pi H^2 + 6\Gamma}{3H^2(1+\omega)^2} - 1 \right) \left(1 - \frac{\Gamma}{3H} \right) + \frac{(1+3\omega^0)}{H_0} \right]. \quad (59)$$

Therefore, we find an expression for the fluctuations of the dominated fluid entropy which are due to the gravitationally induced particle production process. Finally, we should note that since the thermal fluctuation theory allows to evaluate S^2 up to the desired order [79], one can use this equation to find a general relation for the particle production rate Γ , meeting Eq. (49), in a compatible way with thermal fluctuation theory. Indeed, our study is available whenever the state parameter of the dominated perfect fluid be constant. Loosely speaking, in cases with non-constant state parameter $dS^1 \neq 0$ and one should use (43), just the same as the section III, to find an expression for the thermal fluctuations of system up to the desired order. Such situations are appearing in transition eras whiles, the dominated perfect fluid is slowly replaced by the new one and therefore, the state parameter ω is not a constant quantity. We should note again that since one can consider a dominated perfect fluid with constant state parameter ω during an expansion era, between the two transition eras [1], $dS^1 = 0$ is valid in this situation meaning that the gravitationally induced particle production process induces weak fluctuations into the system. These fluctuations are of the second and higher orders which may explain that why this model preserves its final thermodynamic equilibrium state [24, 25].

V. GRAVITATIONALLY PARTICLE PRODUCTION MAY MODIFY THE HORIZON ENTROPY

Here, we are going to study the entropy of apparent horizon of the flat FRW universe, filled by a perfect fluid with state parameter $\omega = \frac{p}{\rho}$, which is also affected by an additional pressure p_C due to the gravitationally induced particle production process [23]. In fact, since it is shown that the DE candidate may add an additional term to the Bekenstein entropy of the horizon [19–21], we are going to investigate the probable effects of the gravitationally induced particle production process, as the DE candidate, on the entropy of the apparent horizon. Moreover, since we work on the apparent horizon, we use the Hayward-Kodama temperature which is in fact an obvious generalization of the Black Holes temperature to the apparent horizon of the FRW universe [21, 40–43]. From the Friedmann equation (47) and (50) we get

$$2HdH = \frac{8\pi}{3}d\rho, \quad (60)$$

and

$$d\rho = -3H(\rho + p)dt - Qdt, \quad (61)$$

respectively. Combining these equations together to reach

$$2HdH + \frac{8\pi}{3}Qdt = -8\pi H(\rho + p)dt. \quad (62)$$

Bearing the Hayward-Kodama temperature ($T = -\frac{H}{2\pi}(1 + \frac{\dot{H}}{2H^2})$) in mind [40–43], we can rewrite this equation as

$$T(-2HdH - \frac{8\pi}{3}Qdt) = -4H^2(\rho + p)dt - 2(\rho + p)dH. \quad (63)$$

Since $E = \rho V$ and $dV = -\frac{4\pi}{H^4}dH$, we get $dE = -4\pi\rho H^{-4}dH - 4\pi H^{-2}(\rho + p)dt$, whiles E is the associated energy due to the source ρ confined to the apparent horizon. By inserting these relations into Eq. (63), we obtain

$$T(-2HdH - \frac{8\pi}{3}Qdt) = \frac{H^4}{\pi}dE + 2(\rho - p)dH, \quad (64)$$

leading to

$$T(-\frac{2\pi}{H^3}dH - \frac{8\pi^2}{3H^4}Qdt) = dE - WdV, \quad (65)$$

where $W = \frac{p-p}{2}$ is the work density [41–44, 46, 47]. Comparing this equation with the first law of thermodynamics in cosmological setups ($TdS_A = dE - WdV$), one gets

$$dS_A = -\frac{2\pi}{H^3}dH - \frac{8\pi^2}{3H^4}Qdt, \quad (66)$$

which yields

$$S_A = \frac{A}{4} + \pi \int \frac{(1+\omega)\Gamma}{H^2}dt + B, \quad (67)$$

where B is the integration constant, $Q = -\rho(1+\omega)\Gamma$, and we have used the Friedmann equation (47) to obtain the second term of RHS of this equation. Using (61) together with (47), one can rewrite the second term of RHS of this equation as

$$\pi \int \frac{(1+\omega)\Gamma}{H^2}dt = \frac{3}{8} \int \frac{\Gamma d\rho}{\rho^2(\Gamma - \sqrt{24\pi\rho})}. \quad (68)$$

Therefore, based on this equation, the DE candidate (gravitationally induced particle production process), may add an additional term to the horizon entropy which leads to modify the Bekenstein limit ($S_A = \frac{A}{4}$). Previously, it has been shown that the dynamic candidates of DE, such as the ghost dark energy model and its generalization, may modify the horizon entropy [19–21]. Moreover, it is shown that any mutual interaction between the cosmos sectors may lead to modify the Bekenstein limit of the horizon entropy [21]. Therefore, the additional term appearing in this equation is in line with the mentioned attempts [19–21]. It is also useful to note that in the absence of a mutual interaction between the gravitational and baryonic fields ($\Gamma = 0$), the results of the previous works are recovered as a desired expectance [21, 40–47].

VI. SUMMARY AND CONCLUDING REMARKS

In this study, we have investigated the thermodynamics of the cosmoses with the initial and final de-Sitter space-times. With this aim, we have showed that the state parameter of varying vacuum, as the DE candidate with varying energy density, cannot be equal to -1 , whenever there is no mutual interaction between the fluids supporting the FRW background except whiles the universe is in the de-Sitter stages with $\dot{H} = 0$ meaning that $\dot{\rho} = 0$. Thereinafter, we have derived an expression for the entropy changes of the introduced DE model. In addition, we have paid our attention to a universe in which the cosmos sectors interact with each other, and we got an entropy change relation due to this interaction. In continue, by focusing on the model with $\omega_D = -1$, we have found its corresponding thermodynamical interpretation. In fact, our study shows that the mutual interaction between the varying vacuum and the other parts of cosmos which disturbs the initial de-Sitter phase of the universe (whiles $\dot{H}_0 = 0$) [22], may also disturb the thermodynamic equilibrium of initial de-Sitter spacetime (where $dS_D^0 = d^2S_D^0 = 0$), and leads to $dS_D \neq 0$. Thereinafter, since this interaction brings the ultimate de-Sitter spacetime (where again $\dot{H} = \dot{H}_0 = 0$) [22] which satisfies again the thermodynamic equilibrium conditions [24], $\dot{\rho}_D = 0$ and thus $Q = 0$. The latter means that $dS_D = dS_D^0$ and thus, the thermodynamic equilibrium conditions are marginally satisfied in the ultimate de-Sitter spacetime ($dS_D^0 = d^2S_D^0 = 0$) which is in agreement with [24]. Therefore, it seems that this mutual interaction between the initial and final de-Sitter eras may leave some thermal fluctuations into the system. Finally, by bearing the thermal fluctuation theory in mind, we got a relation between the thermal fluctuations of the DE candidate and the mutual interaction between the cosmos sectors. Moreover, we have pointed to the models in which there is a particle production process induced by the gravity. We showed that such process may be considered as an interacting model whiles this interaction comes from the gravitationally induced particle process. In fact, our resolution shows that such particle production, coming from non-equilibrium thermodynamic analysis, inspires a weak fluctuation to the thermodynamical properties of the model, including the entropy and temperature, and thus the spacetime features such as its apparent horizon. We have found out a relation between the rate of the particle production and these fluctuations which are second order onwards. The latter may explain why in this model the thermodynamic equilibrium state of cosmos does not change due to the gravitationally induced particle production process [24, 25]. In addition, we have tried to establish the first law of thermodynamics on the apparent horizon and therefore, we found that the gravitationally induced particle production process, as the DE candidate, may add an additional term to the Bekenstein entropy of the apparent horizon. The latter is in line with the previous works claiming that the DE candidate and its interaction with the other parts of cosmos may change the horizon entropy [19–21].

Acknowledgments

This work has been supported financially by Research Institute for Astronomy & Astrophysics of Maragha (RIAAM).

-
- [1] M. Roos, *Introduction to Cosmology* (John Wiley and Sons, UK, 2003).
 - [2] N. Riazi, H. Moradpour and A. Amiri, Prog. Theor. Phys. **126**, 6 (2011).
 - [3] N. Radicella and D. Pavón, Gen. Relativ. Grav. **44**, 685 (2012).
 - [4] S. W. Allen et al., Mon. Not. R. Astron. Soc. **383**, 879 (2008).
 - [5] M. Hicken et al. Astrophys. J. **700**, 1097 (2009).
 - [6] R. Amanullah et al., Astrophys. J. **716**, 712 (2010).
 - [7] E. Komatsu et al., Astrophysical Journal Suplem. **192**, 18 (2011).
 - [8] N. Suzuki et al., Astrophys. J. **746**, 85 (2012).
 - [9] G. Veneziano, Nucl. Phys. B **159**, 213 (1979).
 - [10] A. Sheykhi, Phys. Lett. B **680**, 113 (2009).
 - [11] A. Sheykhi, Class. Quantum Gravit. **27**, 025007 (2010).
 - [12] A. Sheykhi, Phys. Lett. B **681**, 205 (2009).
 - [13] K. Karami, et al., Gen. Relativ. Gravit. **43**, 27 (2011).
 - [14] M. Jamil and A. Sheykhi, Int. J. Theor. Phys. **50**, 625 (2011).
 - [15] A. Sheykhi and M. Jamil, Phys. Lett. B **694**, 284 (2011).
 - [16] S. Nojiri and S. D. Odintsov, Phys. Rept. **505**, 59 (2011).
 - [17] A. R. Zhitnitsky, Phys. Rev. D **86**, 045026 (2012).
 - [18] A. G. Valent, J. Solà and S. Basilakos, JCAP, **01**, 004 (2015).
 - [19] C. J. Feng, X. Z. Li and X. Y. Shen. Mod. Phys. Lett. A **27**, 1250182 (2012).
 - [20] A. Sheykhi, Can. J. Phys. **92**, 529 (2014).
 - [21] H. Ebadi and H. Moradpour arXiv:1505.01030.
 - [22] J. A. S. Lima, S. Basilakos and J. Solà, Mont. Not. R. Astron. Soc. **431**, 923 (2013).
 - [23] J. A. S. Lima, S. Basilakos and F. E. M. Costa, Phys. Rev. D **86**, 103534 (2012).
 - [24] J. P. Mimoso and D. Pavón, Phys. Rev. D **87**, 047302 (2013).
 - [25] J. A. S. Lima, L. L. Graef, D. Pavón and S. Basilakos. JCAP. **2014**, 10 (2014).
 - [26] S. Chakraborty and S. Saha. Phys. Rev. D **90**, 123505 (2014).
 - [27] S. Fay. Phys. Rev. D **89**, 063514 (2014).
 - [28] E. L. D. Perico, J. A. S. Lima, S. Basilakos and J. Sola, Phys. Rev. D **88**, 063531 (2013).
 - [29] L. L. Graef, F. E. M. Costa and J. A. S. Lima. Phys. Lett. B **728**, 400 (2014).
 - [30] L. P. Chimento, M. G. Richarte and Garcia. IvanE. Sanchez, Phys. Rev D **88**, 087301 (2013).
 - [31] J. A. S. Lima and I. Baranov. Phys. Rev. D **90**, 043515 (2014).
 - [32] K. Kleidis and N. K. Spyrou. arXiv:1411.6789 (2014).
 - [33] A. Sheykhi, B. Wang and R. G. Cai, Nucl. Phys. B **779** (2007) 1.
 - [34] A. Sheykhi, B. Wang and R. G. Cai, Phys. Rev. D **76** (2007) 023515.
 - [35] S. A. Hayward, S. Mukohyana and M. C. Ashworth, Phys. Lett. A **256**, 347 (1999).
 - [36] S. A. Hayward, Class. Quantum Grav. **15**, 3147 (1998).
 - [37] B. Wang, Y. Gong and E. Abdalla, Phys. Rev. D **74**, 083520 (2006).
 - [38] Y. Gong, B. Wang and A. Wang, Phys. Rev. D **75**, 123516 (2007).
 - [39] Y. Gong, B. Wang and A. Wang, JCAP, **01**, 024 (2007).
 - [40] D. Bak and S. J. Rey, Class. Quantum Grav. **17**, L83 (2000).
 - [41] P. Binétruy and A. Helou, arXiv:1406.1658v1.
 - [42] D. W. Tian and I. Booth, arXiv:1411.6547v2.
 - [43] A. Helou, arXiv:1502.04235v1.
 - [44] R. G. Cai and S. P. Kim, JHEP **0502**, 050 (2005).
 - [45] R. G. Cai, L. M. Cao and Y. P. Hu, Class. Quant. Grav. **26**, 155018 (2009).
 - [46] M. Akbar and R. G. Cai, Phys. Rev. D **75**, 084003 (2007).
 - [47] R. G. Cai and L. M. Cao, Phys. Rev. D **75**, 064008 (2007).
 - [48] B. Wang, E. Abdalla and R. K. Su, Phys. Lett. B **503**, 394 (2001).
 - [49] B. Wang, E. Abdalla and R. K. Su, Mod. Phys. Lett. A **17**, 23 (2002).
 - [50] R. G. Cai and Y. S. Myung, Phys. Rev. D **67**, 124021 (2003).
 - [51] R. G. Cai and L. M. Cao, Nucl. Phys. B **785** (2007) 135.
 - [52] A. Sheykhi, JCAP. **05**, 019 (2009).
 - [53] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).
 - [54] M. Li, X. D. Li, S. Wang and Y. Wang, Commun. Theor. Phys. **56**, 525 (2011).
 - [55] S. Capozziello and V. Faraoni, Fundamental Theories of Physics. **170**, *Beyond Einstein Gravity* (Springer, NY, 2011).
 - [56] A.G. Riess, et al., Astron. J. **116**, 1009 (1998).

- [57] S. Perlmutter, et al., *Astrophys. J.* **517**, 565 (1999).
- [58] S. Perlmutter, et al., *Astrophys. J.* **598**, 102 (2003).
- [59] P. de Bernardis, et al., *Nature*. **404**, 955 (2000).
- [60] H. Moradpour, A. Sheykhi, N. Riazi and B. Wang, *AHEP*. **2014**, 718583 (2014).
- [61] H. Moradpour and N. Riazi, *Int. J. Theor. Phys.* DOI 10.1007/s10773-015-2659-2.
- [62] N. Radicella and D. Pavón, *Phys. Lett. B* **09**, 031 (2011) 51.
- [63] S. Nojiri and S. D. Odintsov, *Phys. Rev. D* **70**, 103522 (2004).
- [64] I. H. Brevik, S. Nojiri and S. D. Odintsov, *Phys. Rev. D* **70**, 043520 (2004).
- [65] A. A. Costa, X. D. Xu, B. Wang, E. G. M. Ferreira and E. Abdalla, *Phys. Rev. D* **89**, 103531 (2014).
- [66] X. D. Xu, B. Wang and E. Abdalla, *Phys. Rev. D* **85**, 083513 (2012).
- [67] J. H. He, B. Wang and E. Abdalla, *Phys. Rev. D* **83**, 063515 (2011).
- [68] S. Wang, Y. Z. Wang, J. J. Geng and X. Zhang, *Eur. Phys. J. C* **74**, 3148 (2014).
- [69] J. H. He, B. Wang, E. Abdallab and D. Pavón, *JCAP*, **12**, 022 (2010).
- [70] E. Abdalla, L. R. Abramo and J. C. C. de Souza, *Phy. Rev. D* **82**, 023508 (2010).
- [71] X. D. Xu, B. Wang, P. Zhang and F. A. Barandela, *JCAP*, **12**, 001 (2013).
- [72] L. Amendola, *Phys. Rev. D* **62**, 043511 (2000).
- [73] L. Amendola and C. Quercellini, *Phys. Rev. D* **68**, 023514 (2003).
- [74] L. Amendola, S. Tsujikawa and M. Sami, *Phys. Lett. B* **632**, 155 (2006).
- [75] S. del Campo, R. Herrera and D. Pavón, *Phys. Rev. D* **78**, 021302 (2008).
- [76] C. G. Bohmer, G. Caldera-Cabral, R. Lazkoz and R. Maartens, *Phys. Rev. D* **78**, 023505 (2008).
- [77] S. Chen, B. Wang and J. Jing, *Phys. Rev. D* **78**, 123503 (2008).
- [78] D. Pavon and W. Zimdahl, *Phys. Lett. B* **628**, 206 (2005).
- [79] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Oxford, Pergamon, 1969).
- [80] S. Das, P. Majumdar and R. K. Bhaduri, *Class. Quantum Grav.* **19**, 2355 (2002).
- [81] D. Pavón and B. Wang, *Gen. Relativ. Gravit.* **41**, 1 (2009).
- [82] J. Zhou, B. Wang and D. Pavón, *Mod. Phys. Lett. A* **24**, 1689 (2009).
- [83] A. Sheykhi and M. R. Setare, *Mod. Phys. Lett. A* **26**, 1897 (2011).
- [84] K. Karami, et al., *Euro. Phys. Lett.* **93**, 69001 (2011).
- [85] M. Honarvayan, A. Sheykhi and H. Moradpour, *Int. J. Mod. Phys. D* **24**, 1550048 (2015).
- [86] H. Ebadi and H. Moradpour, arXiv:1504.03186.
- [87] M. Maggiore, L. Hollenstein, M. Jaccard and E. Mitsou, *Phys. Lett. B* **704**, 102 (2011).
- [88] J. Sol'a, *Jour. Phys. Conf. Seri.* **283**, 012033 (2011).
- [89] I. L. Shapiro and J. Sol'a, *JHEP*. **0202**, 006 (2002).
- [90] J. Granda, J. Sol'a and H. Stefancic, *Phys. Let. B* **12**, 040 (2006).
- [91] J. Granda, J. Sol'a and H. Stefancic, *JCAP*. **0608**, 011 (2006).
- [92] M. Maggiore, *Phys. Rev. D* **83**, 063514 (2011).
- [93] V. Sahni and A. Starobinsky, *Int. J. Mod. Phys. D9*, 373 (2000). Its citations and references therein.
- [94] H. B. Callen, *Thermodynamics and Introduction to Thermostatistics* (New York: John Wiley and Sons, 1985);
H. B. Callen, *Thermodynamics* (J. Wiley, NY, 1960).
- [95] M. O. Calvão, J. A. S. Lima and I. Waga, *Phys. Lett. A* **162**, 223 (1992).
- [96] I. Prigogine, J. Geheinau, E. Gunzig and P. Nardone, *Gen. Relativ. Gravit.* **21**, 767 (1989).
- [97] J. A. S. Lima and A. S. M. Germano, *Phys. Lett. A* **170**, 373 (1992).
- [98] W. Zimdahl, D. J. Schwarz, A. B. Balakin and D. Pavón, *Phys. Rev. D* **64**, 063501 (2001).
- [99] M. P. Freaza, R. S. de Souza and I. Waga, *Phys. Rev. D* **66**, 103502 (2002).
- [100] S. Basilakos and M. Plionis, *Astron. Astrophys.* **507**, 47 (2009).